Non-linear membrane finite-element analysis for lightweight structure enveloppe design

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Lightweight and textile structures can be modelized by means of the small strains and great displacements non-linear membrane model. Two kinds of finite-element solvers, named explicit and implicit, have been implemented in a software program for a PC computer. Numerical tests and results applied to sail design are presented.

FINITE-ELEMENT METHOD MEMBRANE NON-LINEAR ELASTICITY EQUILIBRIUM SHAPE STRESS MAP SAIL

INTRODUCTION

Two kinds of mechanical models are currently used for lightweight structure analysis (Malinowsky,1993). For a given state of internal stress, and a given external load, the first model gives the resulting equilibrium shape of the membrane structure. It is named the form finding method (Schek,1974). Afterwards a strain/stress analysis can be made using the equilibrium shape previously defined (Mollaert,1991). The second model simultaneously gives the equilibrium shape and the stress map of a membrane structure under a given load and a given initial geometry. By means of the finite-element method, the non-linear great displacements and small strains membrane model equation is solved (Haug and Powell,1971). This second approach has been developed by several authors. Three membrane finite-elements are available:

- The constant stress flat three node triangular element of Oden and Sato(1967).
- The non-constant stress bilinear warped four node quadrilateral element of Haug and Powell(1971).
- The six node quadratic element of D'Uston(1987).

In sail design, the Oden-Sato element is generaly used(Schoop,1990) ,(Jackson,1985). The Haug-Powell element is used by Muttin(1990) for its more high finite-element convergence order. Recently, many numerical tools based on membrane theory, have been developed to investigate mechanical problems such as the deep-drawing process, or vehicle crash simulation. Two kinds of numerical schemes based on the finite-element method can be found. The implicit method solves, by using the Newton method, the non-linear finite-element equilibrium equations written in term of nodal displacement. The first numerical models for sail design used the above method (Jackson,1985), (Muttin,1990), (Schoop,1990). The explicit method, describes the structure behavior, under a given load, as time-dependent (Belytschko,1984). In the recent developments of the explicit methods, some explicit schemes have been used for the calculation of sails (Fukasawa,1993). This publication presents an evaluation of both methods for sail structural analysis.

NUMERICAL MODELS

Implicit model

In previous works (Muttin,1990),(Muttin,1991), we have already used the implicit finite-element scheme of Haug and Powell(1971). The discrete formulation of the membrane model gives a set of non-linear equations, written in term of nodal displacement U:

$$F_{int}(U) = F_{aero} \tag{1}$$

where F_{int} are the nodal forces resulting from internal stress, and F_{aero} are the nodal forces equivalent to external loading (normal wind pressure). Eq. (1) is solved using the Newton-Raphson method. According to an initial zero displacement U_o , the sequel U_i is found as follows:

$$U_o = 0 \quad \text{given} \tag{2}$$

$$U_{i+1} = U_i - \frac{\Delta U_i}{\|\Delta U_i\|_\infty} Max(\|\Delta U_i\|_\infty, \overline{\Delta U}) \tag{3}$$

$$\frac{dF_{int}}{dU}(U_i)\Delta U_i = F_{int}(U_i) - F_{aero} \tag{4}$$

This procedure is done until a convergence criteria on the out-of-balance forces holds. The limit of this sequel gives the displacement of the structure from its initial state. The displacement limiter $\overline{\Delta U}$ is choosen equal to 0.05 meter. It permits a limitation of the first displacement corrections, which may approach infinity. During the first iterations, the under-evaluation of the internal stress renders the stiffness matrix $\frac{dF_{int}}{dU}$ almost singular.

Explicit model

The physical model is completed with the inertial forces and a virtual damping force. The displacement of the structure under external loading is assumed to be time-dependent (Zienkiewics,1991). The discrete problem is then a set of non-linear second order differential equations:

$$F_{acc}(\ddot{U}) = F_{aero} - F_{int}(U) - F_{vis}(\dot{U}) \tag{5}$$

where F_{acc} is the nodal inertial forces and F_{vis} a nodal viscous force. The vectors $\ddot{U}$ and $\dot{U}$ are respectively the nodal acceleration and the nodal velocity. Eq.(5) is solved using a time integration scheme based on a central finite-difference scheme (Eq.6). Starting at time $t{=}0$, for a small enough given time step dt , the following nodal displacement sequel U_t is computed:

$$U_o = \dot{U}_o = 0 \quad \text{given} \tag{6}$$

$$\ddot{U}_{t+dt} = \frac{U_{t-dt} - 2U_t + U_{t+dt}}{dt^2} \tag{7}$$

$$\dot{U}_{t+dt} = \frac{U_{t+dt} - U_t}{dt} \tag{8}$$

$$M\ddot{U}_{t+dt} = F_{a\acute{e}ro} - c\mathbf{1}\dot{U}_{t+dt} - F_{int}(U_{t+dt}) \tag{9}$$

where M is the lumped mass matrix, c is a fictive viscous coefficient, and **1** is the unit matrix. The iterations stop when $\ddot{U}_{t+dt}$ and $\dot{U}_{t+dt}$ are close to zero, i.e. when the membrane structure reaches its equilibrium static state.

By using a non-zero mass matrix, the dynamic relaxation scheme is obtained. Note that the pure viscous relaxation scheme (M=0) has been investigated by Zienkiewics(1985) for linear problems. We adjust the viscous coefficient c to obtain a hyperbolic regime relatively close to the parabolic one. For sail computation we will use c=100. Note that the harmonic regime is independent with mesh finite-element size.

The time step dt is strong mesh dependent, and is chosen so that the first time steps do not provoke numerical explosion. Using the time step stability investigations of Aberlinc(1992) and Zienkiewics(1991), it is possible to prove, in the one dimensional case (cable structure), that dt must decrease as mesh size decreases. Sail calculations confirm the above ratio in the membrane case.

Because subintegration requires hourglass control technique (Flanagan and Belytschko,1981), it was not used in this study. The classical four node Gauss integration rule is used.

RESULTS

Computer implementation

The above numerical schemes have been implemented by means of the MATLAB(c) interpreted language. The cost of the implementation is reduced by using the preprogramming matrix functions, and graphical capabilities. The data structure, and the matrix computations, required by the finite-element method are equally well adapted to this kind of language. Our computations were performed on a PC computer, using a 80486DX2 microprocessor (66 MHz), with 3 Mb of central memory capacity.

Design of a spinaker

We have computed a slightly rounded spinaker under a given wind pressure. The initial geometry, and the loading, are defined by means of the FABRIC CAD-CAM software of CRAIN. The mechanical problem is solved using 64 finite-elements (Fig.1).

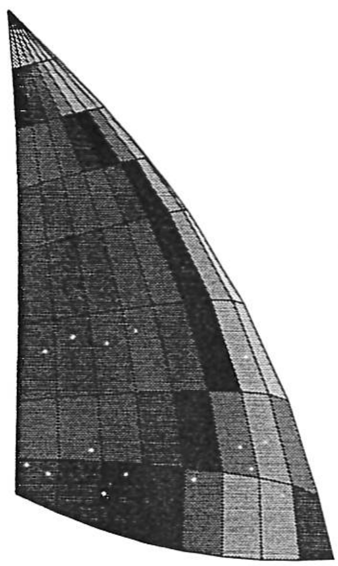


Fig.1 Finite-element mesh.

In assuming the material isotropic with a high Young modulus (1.e+6 N/m), the intrinsic stress field in the sail is obtained. Anisotropic behavior law will result in stress components aligned with the warp and weft directions. The intrinsic stress map permits a design of the textile structure by using the principal stress orientations for fiber directions. The mass matrix is computed using the volumic mass of the structure material (0.15 Kg m-2). The time step used is dt=0.0005 s-1.

Comparison of the implicit and the explicit schemes

For the implicit scheme, the out-of-balance force in terms of the CPU cost is showed in Fig.2. During the first Newton iterations, the residue remains in the same order. During the iterations 5 to 12 the quadratic convergence of the Newton method occurs. After this stage, the residue stays at the precision of the arithmetic processor (1.e-16). The convergence costs 500 CPU seconds.

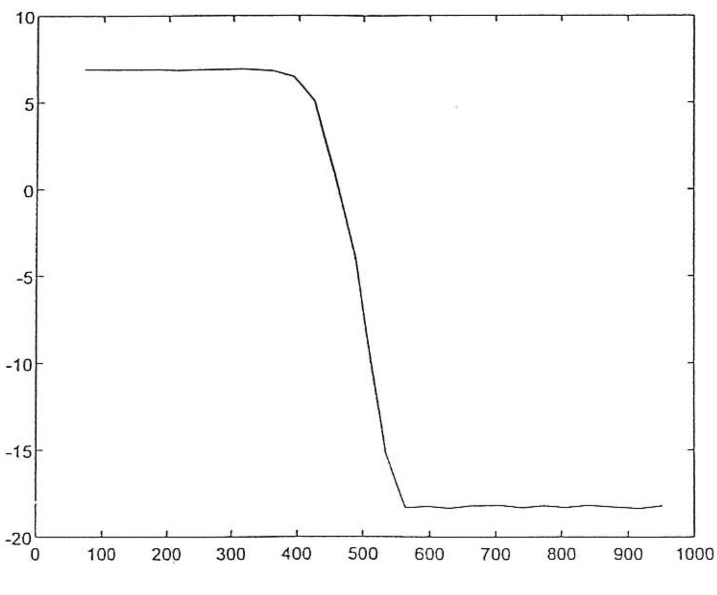


Fig.2 Residual forces in term of CPU cost (implicit scheme).

In Fig.3, the residue in term of CPU cost is shown, for the explicit scheme. During a thousand time steps, using two hours of CPU time, the residue decreases. To reach a convergence criteria, more iterations must be done. In Fig.4, with a coarse 4 by 4 finite-elements mesh, the comparison of the two schemes is presented. As a result of Newton quadratic convergence, it is clear that the implicit scheme is more efficient than the explicit scheme.

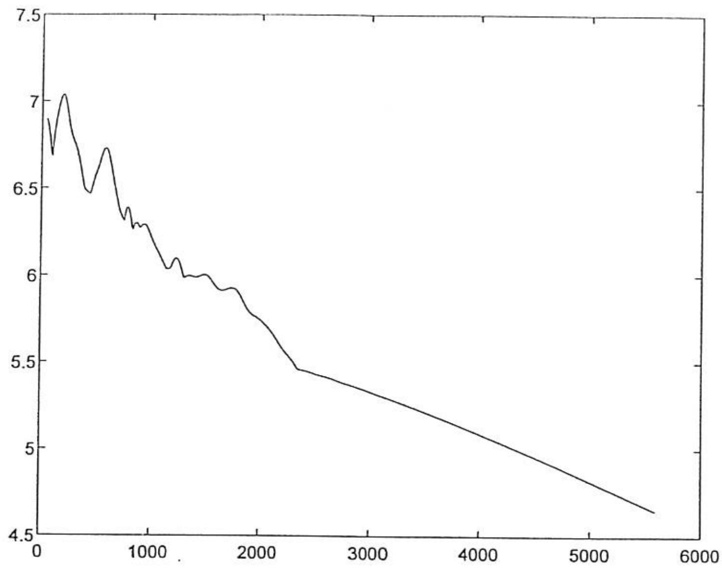
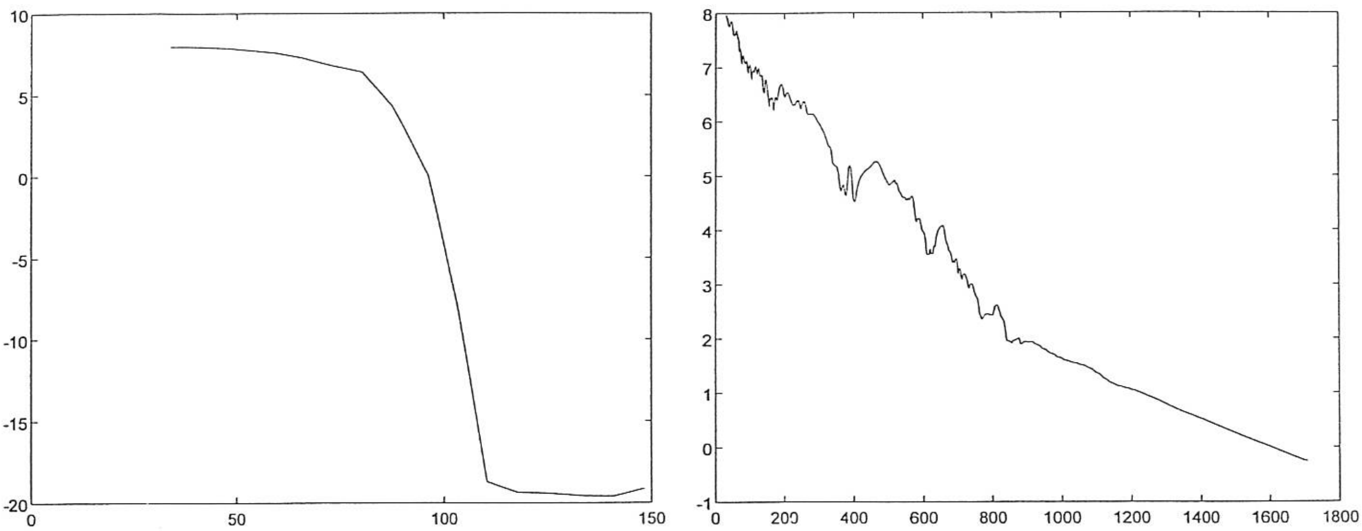


Fig.3 Residual forces in term of CPU cost (explicit scheme).

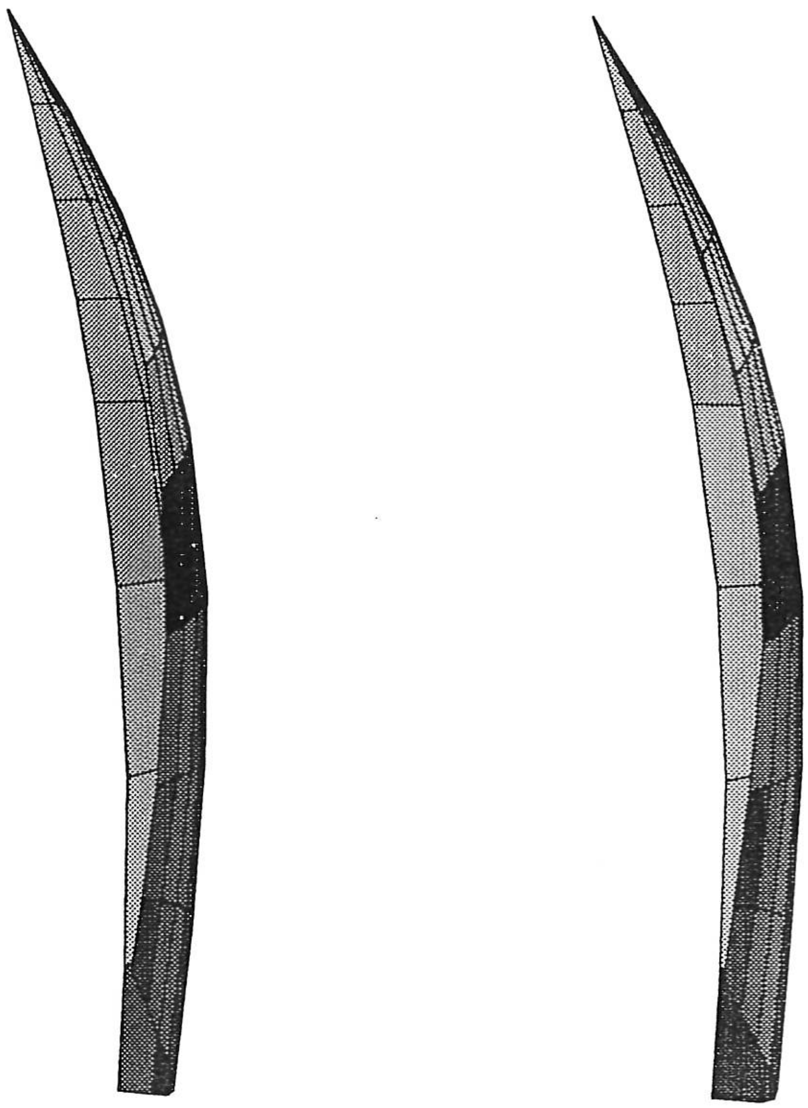


(a)implicit

(b)explicit

Fig.4 Residual forces in term of CPU cost, on a coarse mesh.

In Fig.5, the initial and equilibrium geometries of the sail are presented.



(a)initial (b)final

Fig.5 Initial and equilibrium shapes.

The maximum stress component is showed in Fig.6. The stress concentration at the three nodes of the spi, and the stress flow, can clearly be seen.

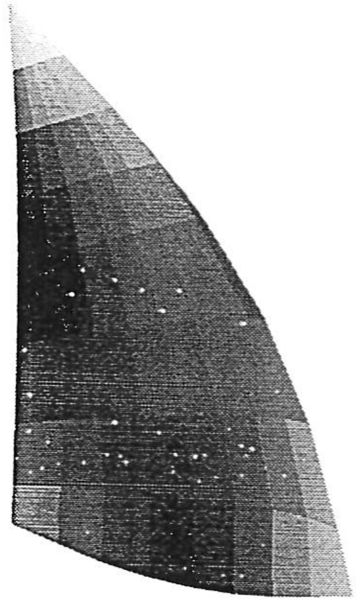


Fig.6 Principal stress flow (dark = law stress).

Fig.7 shows the minimal stress component. Wrinkling region on the spinaker appears where this minimal component is negative.

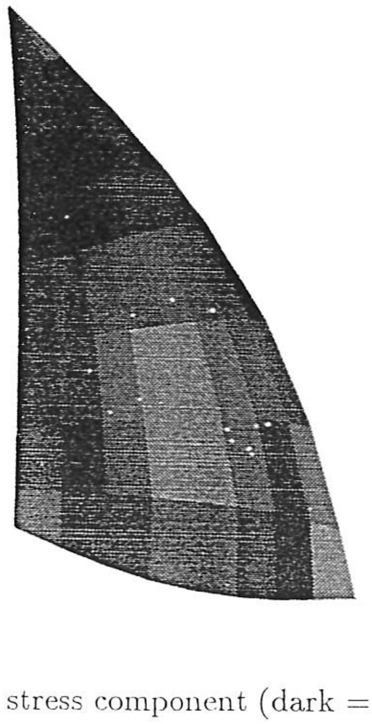


Fig.7 Minimal stress component (dark = law stress).

CONCLUSION

Both explicit and implicit schemes give the equilibrium state of a sail. It appears in the previous computations, that the implicit scheme is less expensive. In the near future we will integrate, cable, beam and contact elements , to treat a range of complex structures including prestressed membranes. Textile and lightweight structures, like LTA (Light Than Air), can be computed using such approach to obtain their equilibrium shape and stress map under pressure loading.

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